Chapter 2

Demand and Supply

This chapter[1] first examines the neoclassical foundation of price adjustment mechanism built on Logical Time, using system dynamics modeling. Then it is argued that similar workings could be done in a real market economy running on Historical Time by the interplay of price, inventory and their interdependent feedback relations. This implies that off-equilibrium analysis built on historical time without neoclassical concept of Auctioneer is a better way of representing market activities. This approach can be one of the foundations of our macroeconomic modeling.

2.1 Adam Smith!

"There's a person who has influenced upon us more than Jesus Christ! Who's he?" An instructor of Economics 1, an introductory course for undergraduate students at the Univ. of California, Berkeley, challenged his students cheerfully. I was sitting in the classroom as a Teaching Assistant for the course. This was in early 80's when I was desperately struggling to unify three schools of economics in my dissertation; that is, neoclassical, Keynesian and Marxian schools of economics.

"He's the author of the Wealth of Nations written in 1776; his name is Adam Smith!", claimed the instructor. Adam Smith's idea of free market economy has been a core doctrine throughout the so-called Industrial Age which started in the middle of the eighteenth century. It has kept influencing our economic life even today with a simple diagram such as Figure 2.1.

Those who have studied economics are very familiar with this diagram of demand and supply, which intuitively illustrates a market mechanism of price

[1]This chapter is based on the paper: Logical vs Historical Time in A Price Adjustment Mechanism in "Proceedings of the 27th International Conference of the System Dynamics Society", Albuquerque, New Mexico, USA, July 26-30, 2009. ISBN 978-1-935056-03-4. It is written during my short-term sabbatical leave at the Victoria Management School, Victoria University of Wellington, New Zealand in March 2009.

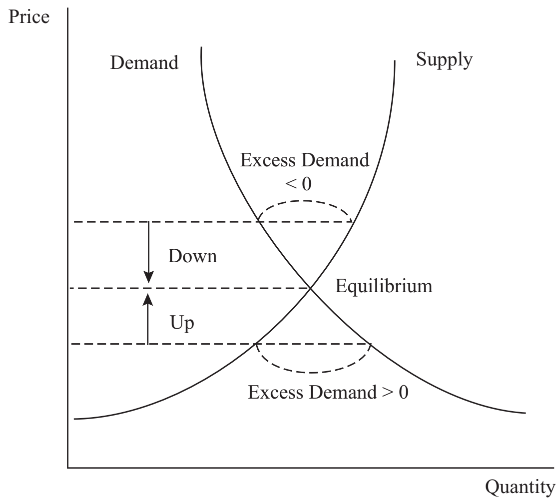

Figure 2.1: Price Mechanism of Demand and Supply

adjustment processes. Price is taken on vertical axis and quantity is taken on horizontal axis. Demand is illustrated as a downward sloping curve, indicating the attitude of consumers that their demand decreases for higher prices and increases for lower prices. This relation is theoretically derived from a utility maximization principle of consumers. Supply is illustrated as an upward sloping curve which exhibits the behavior of producers that their supply increases for higher prices and decreases for lower prices. This relation results from a principle of profit maximizing behavior by producers. Market equilibrium, in which the amount of demand is equal to the amount of supply and market clears, is shown to exist at a point where demand and supply curves intersect in the diagram.

When price is higher than the equilibrium, there exists an excess supply or unsold and increased amount of inventory (which is also called a negative excess demand), and price is eventually forced to go down to attract more consumers to buy the product. On the other hand, if price is lower, there exists an excess demand or the shortage of product which eventually pushes up the price. In either case, price tends to converge to an equilibrium price. This adjusting market force is provided by an *invisible hand*, Adam Smith believed. It is called a price adjustment mechanism, or tâtonnement process, in modern microeconomics.

This price adjustment mechanism works not only in commodity markets but also in labor markets as well as financial capital markets. For instance, let us consider a labor market by taking a wage rate on the vertical axis and the quantity of labor on the horizontal axis. Then, demand curve is interpreted as the demand for labor by producers and supply curve represents the attitude of workers to work. Producers do not employ as many workers as before if wage rate increases, while more workers want to work or they want to work longer hours if their wage rate is higher, and vice versa. Market equilibrium in the labor market denotes full employment. If wage rate is higher than the equilibrium, unemployment comes off and eventually workers are forced to accept a wage cut. In the case of lower wage rate, labor shortage develops and eventually wage rate is pushed up. In this way, price adjustment mechanism works similarly in the labor market.

In a financial capital market, price on the vertical axis becomes an interest rate, and it become a foreign exchange rate in a case of a foreign exchange market. Price mechanism works in a similar fashion in those markets.

In this way, workings of a price adjustment mechanism could be explicated uniformly in all markets by the same framework. Our daily economic activities are mostly related with these market mechanisms governed by the *invisible hand*. This is why the instructor at the UC Berkeley amused his students, saying that Adam Smith has been more influential than Jesus Christ!

Unfortunately, however, this doctrine of *invisible hand*, or neoclassical school of economic thought has failed to obtain unanimous acceptance among economists, and two opposing schools of economics eventually have been struggling to fight against the workings of market price mechanism depicted by Figure 2.1. They are Keynesian and Marxian schools. Mutually-antagonistic dissents of these school created the East-West conflicts, Cold War since the World War II, and domestic right-left wing battle till late 80's when these battles of ideas finally seemed to have ended with a victory of neoclassical school. Since then, the age of the so-called privatization (of public sectors), and globalization with the help of IT technologies have started as if the doctrine of the *invisible hand* has been the robust foundation of free market fundamentalism similar to religious fundamentalisms.

Accordingly most of us believed there would be no longer conflicts in economic thoughts as well as in our real economic life until recently when we were suddenly hit by severe financial crises in 2008; the worst recession ever since the Great Depression in 1929. The battle of ideas seems to be re-kindled against the doctrine of the *invisible hand*. Indeed, the instructor at the UC Berkeley was right. Today Adam Smith seems to be getting more influential globally, not because his doctrine is comprehensive enough to accomplish a consensus on the workings of a market economy, but because it caused many serious socio-economic conflicts and wars instead.

2.2 Unifying Three Schools in Economics

As a graduate student in economics in late 70's and early 80's, I was struggling to answer the question: Why did three schools disagree? As a proponent of Adam Smith's doctrine, neoclassical school believes in a price adjusting mechanism in the market. As shown above, however, this price mechanism only works so long as prices and wages move up and down flexibly in order to attain an equilibrium. Therefore, if disequilibria such as recession, economic crisis and unemployment happen to occur, they believe, it's because economic agents such as monopoly, government and trade unions refuse to accept price and wage flexibility and distort the workings of market mechanism.

Keynesian school considers that market has no self-restoring forces to establish an equilibrium once economic recessions and unemployment occur, because prices and wages are no longer flexible in a modern capitalist market economy. To attain an equilibrium, therefore, government has to stimulate the economy through fiscal and monetary policies. In Figure 2.1 these policies imply to shift the demand curve to the right so that excess supply (and negative excess demand) will be eliminated.

Marxian school believed that market disequilibria such as economic crisis and unemployment are inevitable in a capitalist market economy, and proposed a planned economy as an alternative system. After the collapse of the Soviet Union in 1989, Marxian school ceased to exercise its influence because the experiments of a planned economy in the former socialist countries turned out to be a failure. Even so, they manage to survive under the names of post-Keynesian, environmental economics and institutional economics, etc.

Accordingly, only neoclassical and Keynesian schools remain to continue influencing today's economic policies. In the United States, Republican policies are deeply affected by the doctrine of neoclassical school such as free market economy and small government through deregulation. Meanwhile, Democrats favor for Keynesian viewpoint of public policies such as regulations by wise (not small) government. Current financial crises may reinforce the trend of regulation against hand-free financial and off-balance transactions.

Why do we need three different glasses to look at the same economic reality? Why do we need three opposing tools to analyze the same economic phenomena? These were naive questions I posed when I started studying economics as my profession. In those days I strongly believed that a synthesis of three schools in economics is the only way to overcome Cold War, East-West conflicts and domestic right-left wing battles. By synthesis it was meant to build a unified general equilibrium framework from which neoclassical, Keynesian and Marxian theories can be derived respectively as a special case. My intention was to show that different world views were nothing but a special case of a unified economic paradigm.

While continuing my research toward the synthesis, I was suddenly encountered by a futuristic viewpoint of *The Third Wave* by Alvin Toffler [67]. It was on December 23, 1982, when I happened to pick up the book which was piled up in a sociology section at the Berkeley campus bookstore. The most unimaginable idea to me in the book was the one that both capitalism and socialism were the two sides of the same coin in the industrial age against the leftist doctrine that socialism is an advanced stage of economic development following capitalism. What's an economic system of the Third Wave, then? Can a new economic system in the information age comply with either neoclassical or Keynesian school of economics developed in the industrial age? I kept asking these questions many times in vain, because Toffler failed to present his economic system of the information age in a formal and theoretical fashion.

Being convinced by Toffler's basic idea, however, I immediately decided to develop a simple economic model which could be a foundation of a new economic framework for the information age. In this way, the Third Wave became a turning point of my academic research in economics, and since then my work has been focused on a new economic system of the information age. My effort of synthesizing three schools in economics and creating a future vision of a new economic system fortunately resulted in a publication of the book [78]. Its main message was that three schools in economics are effete in a coming information age, and a new economic paradigm suitable for the new age has to be established.

My idea of economic synthesis was to distinguish Logical Time on which neoclassical school's way of thinking is based, from Historical Time on which Keynesian and Marxist schools of economic thought are based. Yet, the working tools available in those days are paper and pencil. Under such circumstances I was fortunate to encounter by chance system dynamics in middle 90's through the activities of futures studies. Since then, system dynamics modeling gradually started to re-kindle my interest in economics. This chapter examines a true mechanism of the working of market economy, which is made possible by the application of system dynamics modeling.

2.3 Tâtonnement Adjustment by Auctioneer

Let us now construct a simple SD model to examine how a market economy of demand and supply works. In this simple economy buyers and sellers have demand and supply schedules of shirts per week as shown in Table 2.1. These figures are taken from a paper in [76] under the supervision of Professor Jay W. Forrester[2]. The reader can easily replace them with his or her own demand and supply schedules.

Price	Quantity Demanded $D = D(p)$	Quantity Supplied $S = S(p)$
\$ 5	100	0
\$ 10	73	40
\$ 15	57	57
\$ 20	45	68
\$ 25	35	77
\$ 30	28	84
\$ 35	22	89
\$ 40	18	94
\$ 45	14	97
\$ 50	10	100

Table 2.1: Demand and Supply Schedules in [76]

[2]MIT System Dynamics in Education Project (http://sysdyn.clexchange.org/sdep.htm) offers a collection of SD models and papers called Road Maps for self-taught learning of system dynamics. The reader is encouraged to explore these profound resources of SD modeling.

In microeconomics these schedules are called demand and supply functions of market prices and derived rigorously from the axiomatic assumptions of consumers and producers. Demand and supply schedules (or functions $D = D(p)$ and $S = S(p)$) are illustrated in Figure 2.2 in which price is taken on horizontal axis while demand and supply are plotted on vertical axis. This is a standard presentation of functions in mathematics. On the other hand, in standard textbooks of economics price has been traditionally taken on vertical axis as illustrated in Figure 2.1.

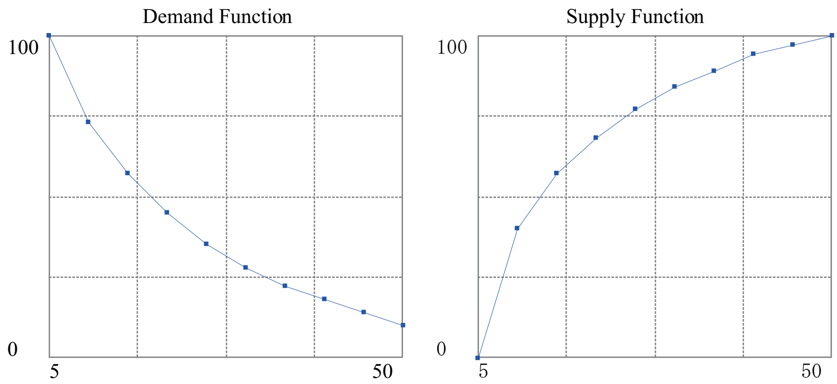

Figure 2.2: Demand and Supply Functions

Now buyers and sellers meet in the market to buy and sell their products according to their schedules of demand and supply. In order to make this market economy work, we need the third player called *Auctioneer* who quotes a price. His role is to raise a price if demand is greater than supply, and lower it if demand is less than supply. His bids continue until the equilibrium is attained where demand is simply equal to supply. This process is called Walrasian or neoclassical price adjustment mechanism or tâtonnement.

The important rule of this market game is that no deal is made until market equilibrium is attained and buyers and sellers can make contracts of transactions. In this sense, time for adjustment is not a real time in which economic activities such as production and transactions take place, but the one needed for calculation. The time of having this nature is called *Logical Time* in [78]. In reality, there are very few markets that could be represented by this market except such as stock and auction markets. Even so, neoclassical school seems to cling to this framework as if it represents many real market transactions.

Equilibrium

Does this market economy work? This question includes two different inquiries: an existence of equilibrium and its stability. If equilibrium does not exist, the

Auctioneer cannot finish his work. If the equilibrium is not stable, it's impossible to attain it. Let us consider the existence problem first.

The Auctioneer's job is to find an equilibrium price at which demand is equal to supply through a process of the above-mentioned tâtonnement or groping process. Mathematically this is to find the price p^* such that

$$D(p^*) = S(p^*) \tag{2.1}$$

In our simple demand and supply schedules in Table 2.1, the equilibrium price is easily found at \$ 15. The existence proof of general equilibrium in a market economy has annoyed economists over a century since Walras. It was finally proved by the so-called Arrow-Debreu model in 1950's. For detailed references, see Yamaguchi [78]. Arrow received Nobel prize in economics in 1972 for his contribution to "general economic equilibrium and welfare theory". He was a regular participant from Stanford University to the Debreu's seminar on mathematical economics when I was in Berkley. Debreu received Nobel prize in economics in 1983 for his contribution to "new analytical methods into economic theory and for his rigorous reformulation of the theory of general equilibrium". I used to attend his seminar on mathematical economics in early 80's, and still vividly remember the day of his winning the prize, followed by a wine party spontaneously organized by faculty members and graduate students.

Stability

The second question is how to find or attain the equilibrium. From the demand and supply schedules given above, there seems to be no difficulty of finding the equilibrium. In reality, however, the Auctioneer has no way of obtaining these schedules. Accordingly, he has to grope them by quoting different prices. To describing this groping process, a simple SD model is built as in Figure 2.3 [Companion model: 1 Auctioneer.vpm].

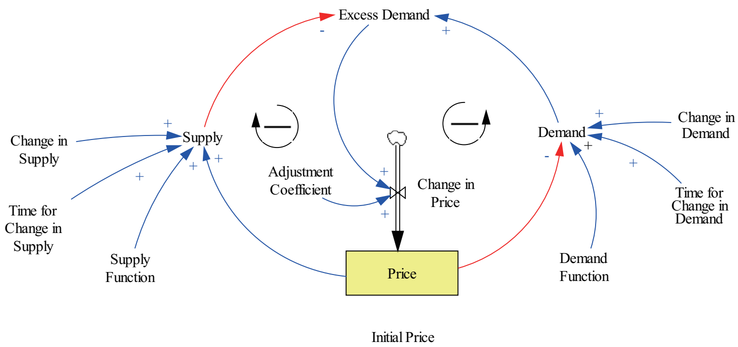

Figure 2.3: Auctioneer's Tâtonnement Model

Mathematically, the model is formulated as follows:

$$\frac{dp(t)}{dt} = f(D(p) - S(p), \lambda) \tag{2.2}$$

where f is excess demand function and λ is a price adjustment coefficient. In the model f is further specified as

$$f = \lambda \, \frac{D(p) - S(p)}{D(p) + S(p)} \, p \tag{2.3}$$

From the simulations in our simple model the idea of tâtonnement seems to be working well as illustrated in Figure 2.4. The left-hand diagram shows that the initial price of \$10 tends to converge to an equilibrium price of \$15. Whatever values of initial price are taken, the convergence can be similarly shown to be attained. In this sense, the market economy can be said to be globally stable. With this global stability, the Auctioneer can start with any quotation of initial price to arrive at the equilibrium successfully.

In the right-hand diagram, demand schedule is suddenly increased by capricious buyers by 20 units at the week of 15, followed by the reactive increase of the sellers in the same amount of supply at the week of 30, restoring the original equilibrium. In this way, the Auctioneer can easily respond to any changes or outside shocks and attain new equilibrium states. These shifts of demand and supply curves are well known in microeconomics as comparative static analysis.

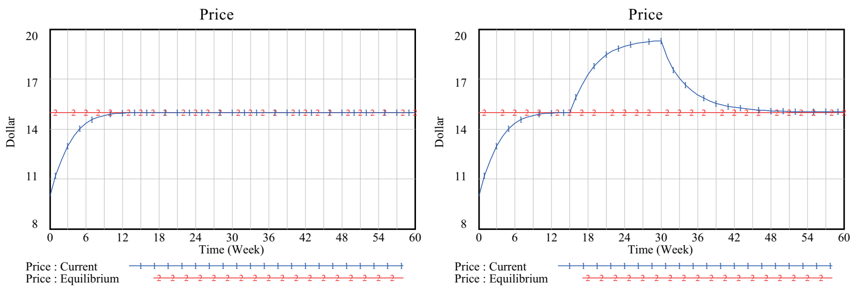

Figure 2.4: Stability of Equilibrium

Chaos

So far, neoclassical price mechanism seems to be working well. To attain the equilibrium in our model, a price adjustment coefficient is set to be 0.4. What will happen if the Auctioneer happens to increase the adjustment coefficient from 0.4 to 3 in order to speed up his tâtonnement process? Surprisingly this has caused a period 2 cycle of price movement with alternating prices between 10.14 and 18.77, as illustrated in the left-hand diagram of Figure 2.5. When the coefficient is increased a little bit further to 3.16, price behavior suddenly becomes very chaotic as the right-hand diagram illustrates. I encountered this chaotic price behavior unexpectedly when I was constructing a pure exchange economic model using S language under UNIX environment in [79].

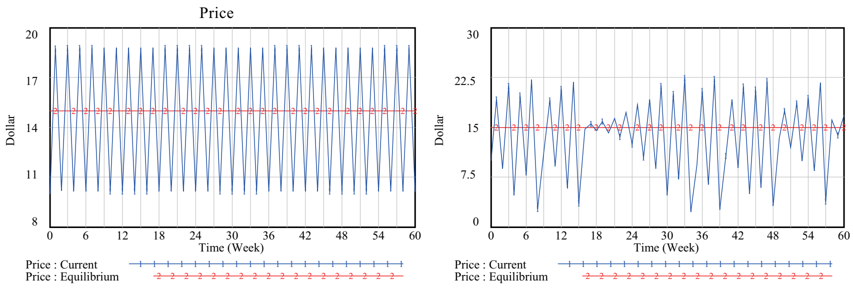

Figure 2.5: Chaotic Price Behavior

Under such a chaotic price behavior, it is obvious that the Auctioneer fails to attain an equilibrium price. Accordingly, under the failure of finding the equilibrium, market transactions can never take place according to the neoclassical rule of the market game. This indicates a fundamental defect in neoclassical framework of market economy based on the idea of *logical* time.

Short-side Transactions

Tired with an endless struggle by the Auctioneer to attain an equilibrium in a chaotic price behavior, buyers and sellers may force their actual transactions to resume at a short-side of demand and supply. In other words, if demand is greater than supply, the amount supplied at that price is traded, while the amount demanded is purchased if supply is greater than demand.

To allow this off-equilibrium transactions, the Auctioneer has to have enough amount of inventory at hand before the market starts. To calculate the enough amount of inventory, a slightly revised model is built as shown in the left-hand diagram of Figure 2.6 [Companion model: 2 Auctioneer(Inventory).vpm].

When the Auctioneer quotes an initial price below equilibrium at $5, allowing the short-side trade, unrealized excess demand keeps piling up as backlog due to an inventory shortage and the amount accumulates up to 325.30 shirts. When market price is initially quoted above equilibrium at $25, excess supply causes inventory of unsold shirts to piles up to 137.86 shirts, as illustrated in the right-hand diagram of Figure 2.6. If the Auctioneer is allowed to have these amount of inventories from the beginning, he could find an equilibrium price even by allowing these inter-auction transactions. Since no shirts are made available until the equilibrium contract is made and production activities start under the neoclassical rule of market game, this short-side off-equilibrium deal is logically impossible. In other words, no feedback loop is made available without inventory from the viewpoint of system dynamics. In conclusion, the existence of chaotic price behavior and neoclassical assumption of market economy are inconsistent.

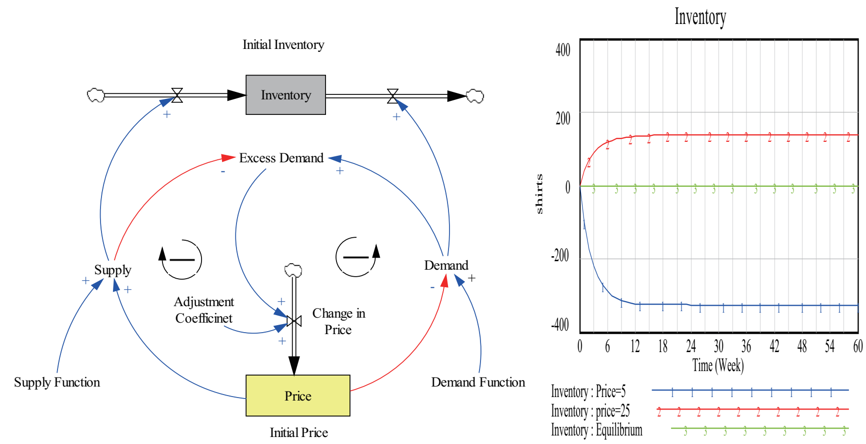

Figure 2.6: Short-side Transaction Model and Inventory

2.4 Price Adjustment with Inventory

The above analysis indicates it's time to abandon the neoclassical framework based on Logical Time. In reality, production and transaction activities take place week by week, and month by month at short-side of product availability, accompanied by piled-up inventory or backlog. Time flow on which these activities keep going is called *Historical Time* in [78]. In system dynamics, demand and supply are regarded as the amount of flow per week, and flow eventually requires its stock as inventory to store products. Thanks to the inventory stock, transactions now need not be waited until the Auctioneer finishes his endless search for an equilibrium. This is a common sense, and even kids understand this logic. In other words, a price adjustment process turns out to require inventory from the beginning of its analysis, which in turn makes off-equilibrium transactions possible on a flow of Historical Time.

This disequilibrium approach is the only realistic method of analyzing market economy, and system dynamics modeling makes it possible. The model running on Historical Time for simulations, which is based on [76], is drawn in Figure 2.7 [Companion model: 3 Inventory.vpm].

Price no longer need to respond to the excess demand, instead it tries to adjust to the gap between inventory and desired inventory. To avoid a shortage under off-equilibrium transactions, producers usually try to keep several weeks of the demanded amount as inventory. This amount is called desired inventory. An inventory ratio is thus calculated as the inventory divided by the desired inventory. And market prices are assumed to respond to this ratio. Table 2.2 specifies the effect of the ratio on price. For instance, if the actual inventory is 20% larger than the desired inventory, price is assumed to be lowered by 25%. Vice versa, if it's 20% smaller, then price is assumed to be raised by 35%.

Mathematically, the model is formulated as follows:

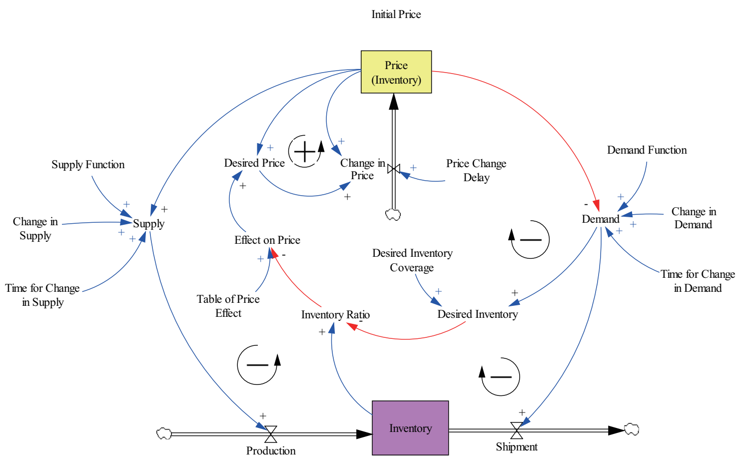

Figure 2.7: Price Adjustment Model with Inventory

Inventory Ratio	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
Effect on Price	1.8	1.55	1.35	1.15	1	0.875	0.75	0.65	0.55

Table 2.2: Effect of Inventory Ratio on Price

$$\frac{dp(t)}{dt} = \frac{p^* - p(t)}{PCD} \tag{2.4}$$

where PCD is a parameter of price change delay, and p^* is a desired price such that

$$p^* = p(t)g\left(\frac{x(t)}{x^*}\right) \tag{2.5}$$

Function g is a formal presentation of the numerical relation in Table 2.2, and $x(t)$ and x^* denote inventory and desired inventory, respectively, such that

$$\frac{dx(t)}{dt} = S(p) - D(p) \tag{2.6}$$

$$x^* = \alpha D(p) \tag{2.7}$$

where α is a parameter of desired inventory coverage[3].

[3]Without using a table function, function g could be mathematically represented as

$$g = \frac{1}{x^e}, \text{ where } x = \frac{x(t)}{x^*} \tag{2.8}$$

Elasticity of the function g can be calculated as

$$\text{Elasticity} \equiv -\frac{dg}{g}/\frac{dx}{x} = -\frac{dg}{dx}\frac{x}{g} = -\left(-\frac{e}{x^{e+1}}\right)\frac{x}{g} = e \tag{2.9}$$

Hence, the function g has a uniform elasticity e over its entire range.

Under such circumstances, the initial price is set here at \$10 as in the case of the Auctioneer's tâtonnement. Price (line 5) now fluctuates around the equilibrium price of \$15 by overshooting and undershooting alternatively, then tends to converge to the equilibrium as illustrated in Figure 2.8. Inventory gap (= desired inventory - inventory) is the gap between line 4 and 3, and price responds to this gap rather than the excess demand (the gap between line 1 and 2). The reader can easily confirm that price tends to rise as long as the inventory gap is positive, or inventory ratio is lower than one, and vice versa.

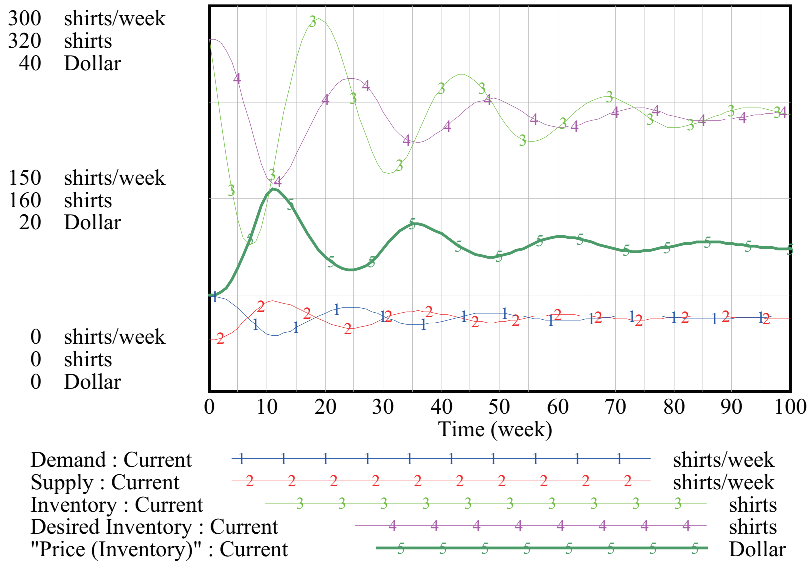

Figure 2.8: Price Adjustment with Inventory

In the left-hand diagram of Figure 2.9, demand is increased by 20 units at the week of 15, followed by the increase in the same amount of supply at the week of 30, restoring the original equilibrium as in the case of the Auctioneer's tâtonnement, though overshooting this time. These shifts of demand and supply curves, however, may no longer be appropriate to be called comparative static analysis method in microeconomics, because we are no longer comparing two different states of equilibrium points. Right-hand diagram illustrates how price cycle is triggered by reducing the original inventory coverage of 4 weeks to 2.3

Desired price p^* can be now rewritten as

$$p^* = p(t)\frac{1}{x^e} \tag{2.10}$$

Price adjustment processes are constructed in this fashion in our macroeconomic models below.

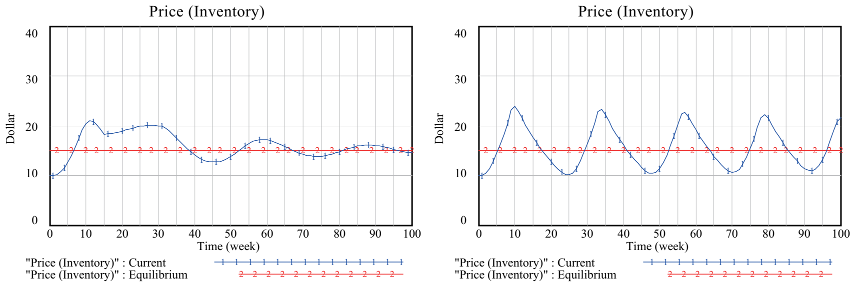

Figure 2.9: Effects of the Changes in Demand, Supply and Inventory Coverage

weeks. In conclusion, system dynamics modeling makes it possible to describe the actual off-equilibrium transactions and price behaviors along the Historical Time.

2.5 Logical vs Historical Time

A combined model is created in Figure 2.10 to compare how the above two price adjustment processes behave differently; one is running on Logical Time and the other on historical time [Companion model: 4 Comparison.vpm].

Left-hand diagram of Figure 2.11 is produced to show similar patterns by setting the Auctioneer's adjustment coefficient to be 2.7. In both cases it takes about 100 weeks to attain the equilibrium. The difference is that under logical time production and transactions never take place until the equilibrium is attained around the Logical Time of 100 weeks, while a real economy running on the Historical Time is suffering from the fluctuation of inventory business cycles for 100 weeks until a real equilibrium price is attained.

What will happen if the demand suddenly increases by 20 at week 50. Right-hand diagram illustrates the real economy can no longer attain the equilibrium in 100 weeks. In this way the market economy is forced to be fluctuating around off-equilibrium points forever in face of continued outside shocks, compared with a quick realization of the equilibrium by the Auctioneer around the Logical Time of week 70.

The meaning of logical and Historical Times is now clear. Microeconomic textbooks are full of Logical Time analyses when dynamics of price movements are discussed. The reader now has the right to ask if the time in textbooks is logical or historical. If historical, price has to be always accompanied by the inventory on Historical Time.

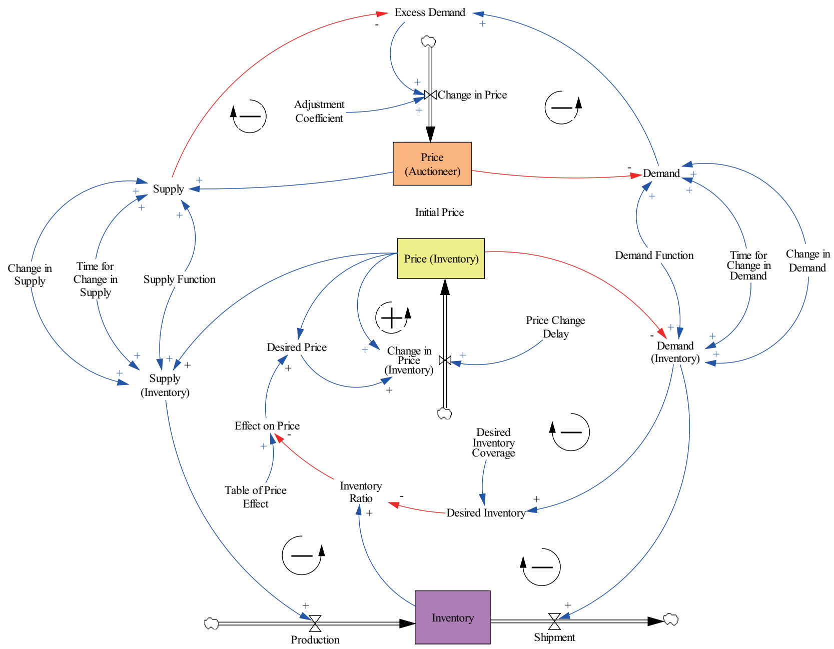

Figure 2.10: Auctioneer vs Inventory Price Mechanism Compared

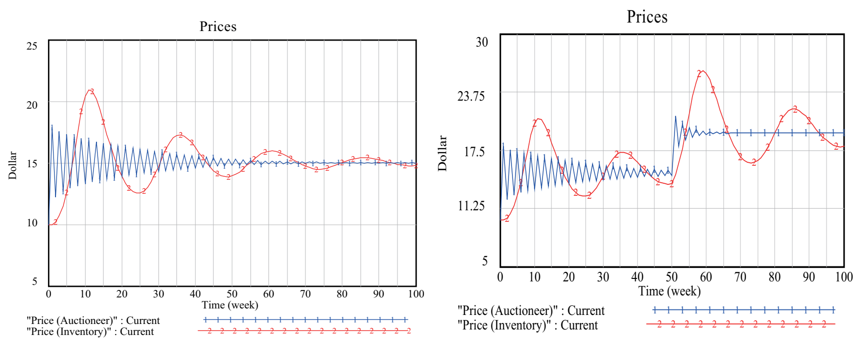

Figure 2.11: Auctioneer vs Inventory Price Behaviors

2.6 Stability on A Historical Time

Which path, then, should we follow to analyze free market economic activities? Neoclassical analysis of Logical Time is mathematically rigorous, yet free price behavior is no longer stable, as preached by market fundamentalists, due to the appearance of Chaos as shown above. In other words, market economy could be chaotic even on the basis of neoclassical doctrine.

On the other hand, analysis running on historical time is off-equilibrium and looks unstable, full of business cycles; that is, chaotic as well. Yet, there's a way to make the historical time analysis stable and free from business cycles. To do so, let us now change the seller's supply (production) schedule so that it can reflect the inventory gap as follows:

$$\begin{aligned} \text{Supply (Inventory)} &= \text{Supply Function (Price (Inventory))} \\ &\quad + \text{Inventory Gap / Inventory Adjustment Time} \end{aligned} \tag{2.11}$$

Mathematically, equation (2.6) is replaced with the following:

$$\frac{dx(t)}{dt} = S^*(p) - D(p) \tag{2.12}$$

$$S^*(p) = S(p) + \frac{x^* - x(t)}{IAT} \tag{2.13}$$

where IAT is inventory adjustment time.

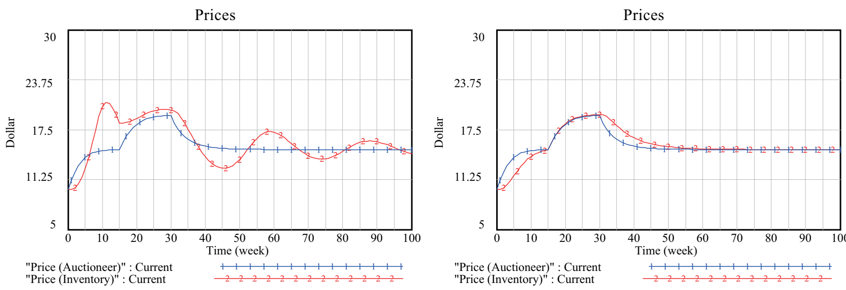

Figure 2.12: Historical Price Stability with Adjusted Supply Schedule (1)

Left-hand diagram of Figure 2.12 illustrates how price behaviors are different between Logical Time (line 1) and historical time (line 2) when demand is increased by 20 units at the week of 15, followed by the increase in the same amount of supply at the week of 30 [Companion model: 5 Comparison(Supply).vpm)]. In both cases prices try to restore the original equilibria, though their speed and meaning are different. In the right-hand diagram, newly adjusted supply schedule is now applied with the inventory adjustment time of 3 weeks. To our surprise, almost the same price behavior is obtained as the one on Logical Time.

In the left-hand diagram of Figure 2.13, price behavior on the Logical Time is illustrated as line 1 for the initial price at \$10, while the same price behavior on the Historical Time is illustrated as line 2 for the inventory coverage of 2.3 weeks, similar to the right-hand diagram of Figure 2.9. Now the new supply

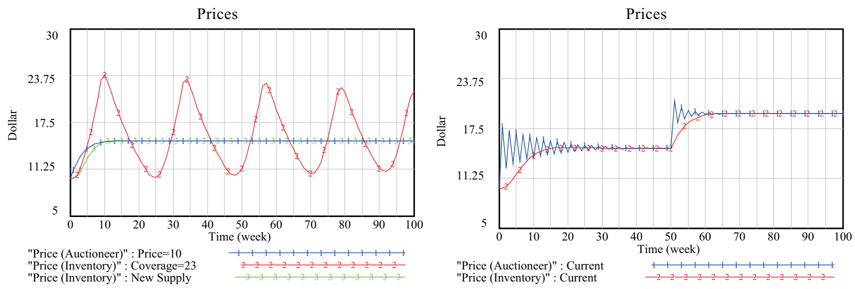

Figure 2.13: Historical Price Stability with Adjusted Supply Schedule (2)

schedule is applied to the same situation, which results in line 3. Again, the line 3 becomes very similar to the price behavior (line 1) on the Logical Time.

Finally let us apply the new supply schedule to the right-hand diagram of Figure 2.11, that is previously explained as the case in which "the real economy can no longer attain the equilibrium in 100 weeks." Right-hand diagram of Figure 2.13 is the result obtained by newly adjusted supply with the inventory adjustment time of 3 weeks. Again almost similar price behavior is restored as the one on the Logical Time.

These simulation results may indicate that our market economy could behave as close as the one predicted by neoclassical equilibrium analysis on Logical Time so long as economic agents behave appropriately on the historical off-equilibrium time. In other words, we no longer need a help from Auctioneer running on logical time to attain an equilibrium in a market economy. Price, inventory and their interdependent feedback relations can do the same job in a real market economy.

2.7 A Pure Exchange Economy

2.7.1 A Simple Model

Chaotic price behavior observed in tâtonnement adjustment is not specific to a partial or single market. To show a Chaos in a general equilibrium framework, let us consider a pure exchange economy: the most favored economy used by neoclassical economists in textbooks. A pure exchange economy is a kind of game without production in which initially endowed goods are exchanged on the basis of traders' own preferences such that their utilities are maximized. Such an exchange economy is profoundly criticized by Joan Robinson [55] as an irrelevant game in a prison camp in which prisoners are given fairly equal amounts of commodities irrespective of their personal tastes so that an exchange game based on their tastes can easily proceed. I have also criticized its appropriateness as a capitalist economic model [78, Chapter 7], and posed a more comprehensive model comprising the analysis of both logical and Historical Time for a better understanding of the functioning of a capitalist market economy[78, Chap.3-6].

Yet, the exchange model is still used in most textbooks on microeconomics as a first approximation to a market mechanism. If there still exists something that we can learn from a pure exchange model, it is the functioning of a tâtonnement price adjustment mechanism. The structure of the price mechanism is basically the same for a more general economy with production. Thus, Hildenbrand and Kirman justify the analysis of a pure exchange economy as saying "if we cannot solve, in a reasonably satisfactory way, the exchange problem, then there is not much hope for the solution of the more general one [35, pp.51-51]." I have indicated [78] that this justification is only applicable to the analysis of *logical* time, but not to that of *historical* time. A pure exchange model should, therefore, be confined to a heuristic use for understanding a price mechanism of Logical Time.

Understanding the exchange economy this way, do we still have unanswered problems? The answer seemed to me to be negative at first, since the economy has been comprehensively studied in the literature, for instance, [35] and [60]. However, there still exist some interesting questions in the area of numerical computations and simulations of price adjustment mechanisms using system dynamics modeling.

The economy is explained as follows. It consists of at least two traders (and consumers simultaneously) who bring their products to the market for exchange. Their products are called *initial endowment* in economics, which becomes the source of supply in the market. We assume following endowment for consumer 1 and 2.

$$\begin{cases} \text{Consumer 1} & = (10,\ 6) \\ \text{Consumer 2} & = (\ 6, 15) \end{cases} \tag{2.14}$$

The economy can thus evade the analysis of production. That's why it is called a pure exchange economy.

In the pure exchange economy only relative prices matter due to the Walras law[4]. Let us assume that commodity 1 becomes a numeráire, that is, its price is unitary: $p_1 = 1$, $p_2 = p$ be a relative price of commodity 2.

When the price is quoted in the economy, traders evaluate a market value of their products as a source of their income for further exchange or purchase of the products in the market. Then as consumers, they try to maximize their utility (which is derived from the consumption of the products purchased in the market) according to their own preferences subject to their income constraint. In this way their demand for products are calculated as a function of prices, income (which in turn is a function of prices) and preferences. Total demand is obtained as a sum of these individual consumer's demand, which is then compared with the total supply. Excess demand is defined as the difference between total demand and total supply, and becomes a function of prices and

[4]See the appendix for detail.

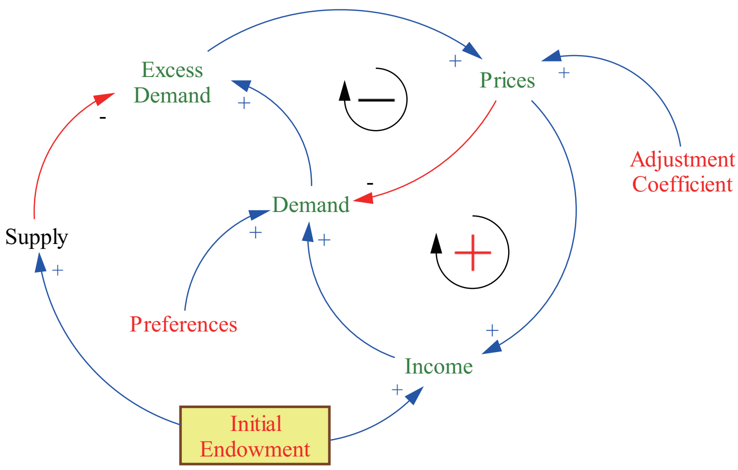

Figure 2.14: A Causal Loop of A Pure Exchange Economy

preferences. Figure 2.14 illustrates a causal loop diagram of the pure exchange economy.

Market prices have two causal loops; one positive and one negative feedback loops. In the figure they are indicated by plus and minus signs. Positive loop in general tries to reinforce the original move stronger, while negative loop tries to counterbalance it. Thus, a moving direction of market prices depends on which loop is dominating: positive or negative? When a positive feedback loop dominates, prices tend to diverge, while a negative feedback loop reverses the direction of the price movement. These opposite and complicated movements are caused by the values of two parameters: adjustment coefficient and preferences. Pure exchange model is illustrated in Figure 2.15 [Companion model: 6 PureExchange.vpm].

2.7.2 Tâtonnement Processes on Logical Time

A step-by-step calculation process of price adjustment is depicted in Figure 2.16, where P_t denotes prices at the period t, a function f denotes the amount of excess demand, and α, λ denotes preferences and price adjustment coefficient, respectively. Preferences and adjustment coefficient are the only parameters in the economy which have to be exogenously determined. Once these are given and present prices are quoted, excess demand can be calculated. If it is positive, prices at the next period are increased by the amount of the excess demand multiplied by the adjustment coefficient. Hence, adjustment coefficient determines the degree of a price increase in the next period. When excess demand is negative, prices at the next period are decreased by the amount of the excess demand multiplied by the adjustment coefficient.

As illustrated in Figure 2.1, a convergence of prices to the equilibrium is

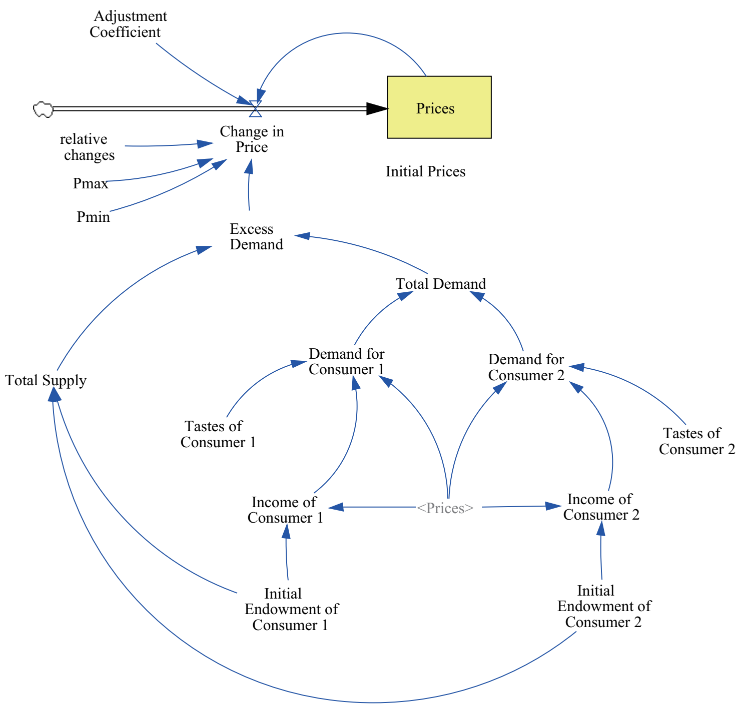

Figure 2.15: A Pure Exchange Economy Model

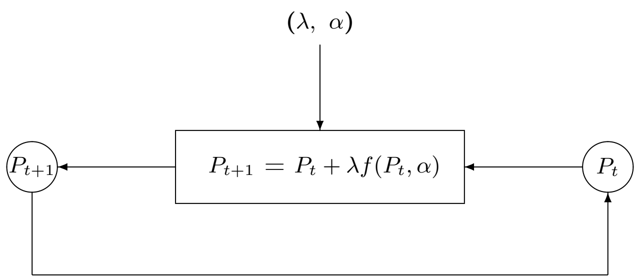

Figure 2.16: Tâtonnement Adjustment Process

expected where demand and supply curves intersect. Indeed, they did for a very small value of adjustment coefficient; that is, prices are shown to be globally stable.

To our surprise, however, something strange happened as the value of the coefficient increased. As Figure 2.17 indicates, the adjustment process begins to produce a clear bifurcation, or an oscillation of prices in period 2 when price

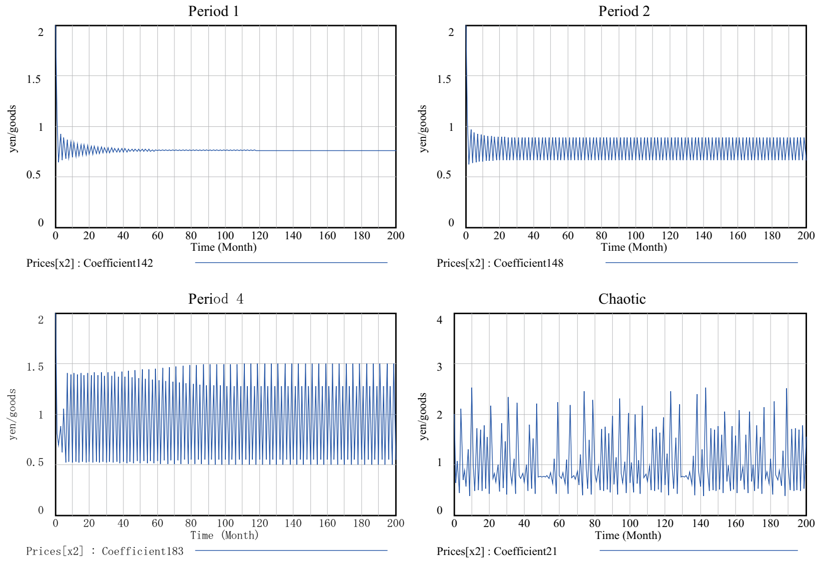

Figure 2.17: Price Movement of Period 1, 2, 4 and Chaos

adjustment coefficient is $\lambda = 0.148$. Furthermore, an increasing adjustment coefficient continues to create new bifurcations or price oscillations of period $2n$, $n = 1, 2, \ldots$ until it became totally chaotic. In other words, instead of converging to an equilibrium or diverging to infinity, market prices seemed to be eventually attracted to a certain region and continue fluctuating in it, with the information of initial values being lost shortly.

Figure 2.18 illustrates the bifurcation of prices as the adjustment coefficient increases. The region is called *a strange attractor* or *Chaos*. Hence, a price mechanism in a market economy turned out to be chaotic! In such a chaotic region, market economy becomes far from the equilibrium and globally unstable, and economic disequilibria such as recession and unemployment become dominant.

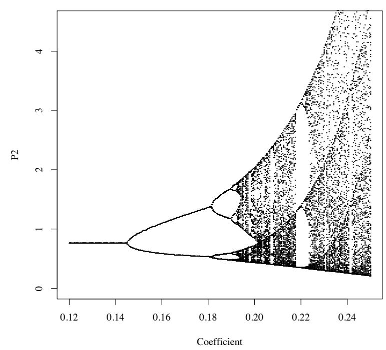

Figure 2.18: Chaotic Price for λ

One of the main features of Chaos is a sensitive dependence on initial conditions. This means that a very small difference of initial values will create a big difference later

on and a long run prediction of the
movement will become eventually impossible. This is confirmed by plotting prices as time-series data. In Figure 2.19 two lines represent time-series behaviors of two prices whose initial values only differ by 0.00001. Line 1 is obtained at $\lambda = 0.21001$, while line 2 is obtained at $\lambda = 0.21002$. Evidently two lines begin to diverge as time passes around month 20, which proves that prices are indeed chaotic (See [79] for details).

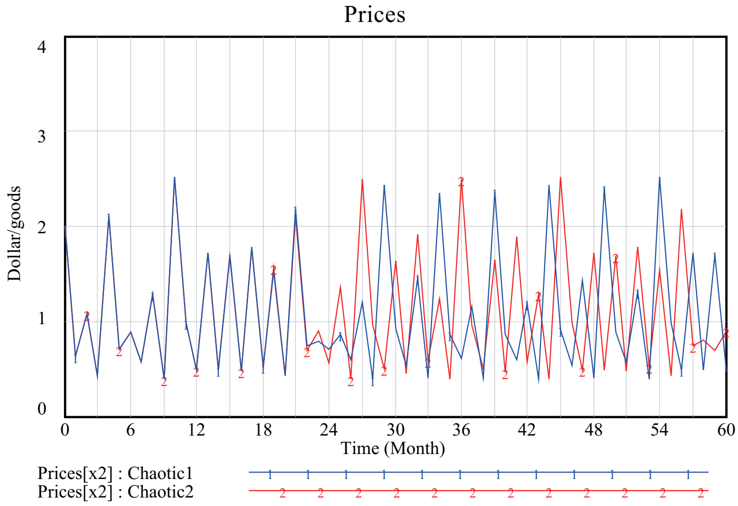

Figure 2.19: Sensitive Dependence on Initial Conditions

It is almost impossible in reality to obtain a true initial value due to some observation errors and round-off errors of measurement and computations. These errors are exponentially magnified in a chaotic market to a point where predictions of future prices and forecasting are almost meaningless and even misleading.

This is a wholly unexpected feature for a neoclassical doctrine of market stability originated by Adam Smith's idea of *invisible hand*. Even so, this chaotic situation could be harnessed so long as the value of adjustment coefficient is small enough; in other words, prices are regulated to fluctuate only within a small range so that no violent jumps of prices are allowed - a relief to the neoclassical school.

2.7.3 Chaos Triggered by Preferences

What will happen, then, if preferences, an another parameter in the economy, vary? Can whimsical preferences of consumers are also powerful enough to drive a stable economy into chaos? To examine this, I started with a globally stable situation in which a price, wherever its initial position is, converges to an equilibrium price; specifically at the adjustment coefficient of $\lambda = 0.138$. Then tastes of goods 1 for consumer 1 is increased from 0.5 slightly up. Figure 2.20 illustrates periodic behavior of price caused by the changes in consumer 1's tastes.

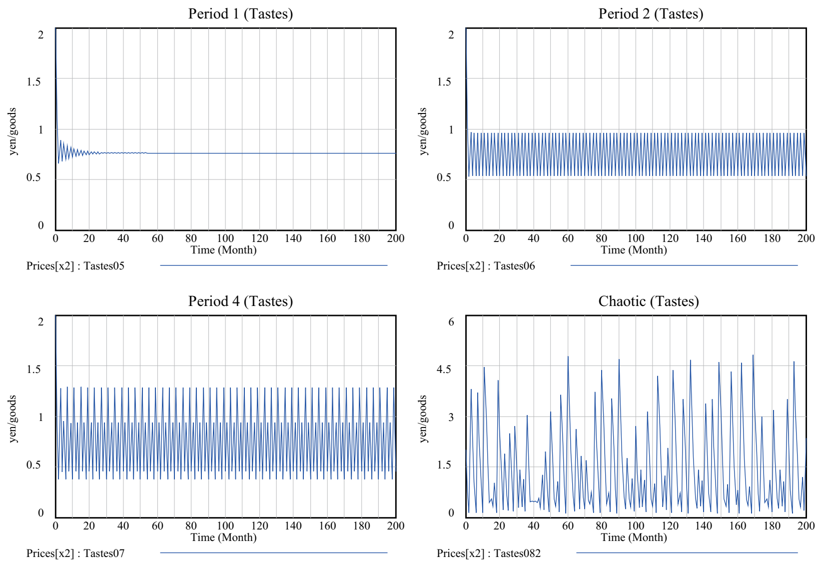

Figure 2.20: Price Movement of Period 1, 2, 4 and Chaos Caused by Tastes

Figure 2.21 is produced by changing the values of preferences α. It indicates that as the values of α increase an equilibrium price tends to be going down up to a bifurcation point. Except this decreasing equilibrium price, to our surprise, both diagrams in Figures 2.18 and 2.21 turned out to be structurally similar; that is, Chaos is similarly caused by the changes in preferences (Fore details see [79] and [80]).

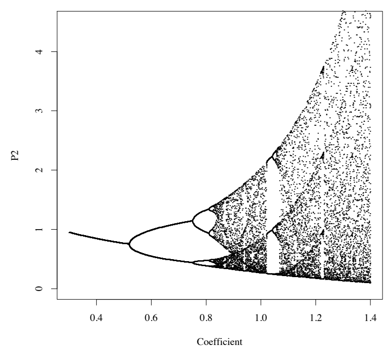

Figure 2.21: Chaotic Price for α

This seems to be a serious challenge against a neoclassical doctrine of price stability. Market equilibrium can no longer be restored even by a small value of adjustment coefficient. That is to say, price regulations suggested above are no longer effective to harness a Chaos in the market. The price stability attained by a small value of adjustment coefficient can be easily driven into a chaos by whimsical preferences of consumers. Capricious behaviors of consumers themselves are the cause of chaos and, to be worse, no regulations are possible to control consumers' preferences. It is concluded, therefore, that Chaos is inherent in the market, to be precise, of Logical Time.

2.7.4 Off-Equilibrium Transactions on Historical Time

What is an economic implication of this chaotic price adjustment, then? Pure exchange economy works only when its Auctioneer can find equilibrium prices at which traders and consumers make their transactions. If the Auctioneer cannot find the equilibrium, market failure arises according to neoclassical framework of market economy. The Auctioneer could become totally helpless in the face of an unpredictability of market prices and the existence of Chaos itself in the market economy.

Chaos is caused by the values of two parameters; adjustment coefficient and preferences. The Auctioneer could find the coefficient value which attains price stability and eventually equilibrium. This could be done by harnessing a chaotic movement of prices, as mentioned above, by imposing a price regulation directly or setting a market rule for price changes. These policies of the Auctioneer inevitably begin to justify a Keynesian school's idea of utilizing public policies by wise government.

Yet, Chaos is triggered by another parameter of consumers' preferences. This time the Auctioneer has no direct or indirect control over preferences and tastes of consumers. This means consumers' whimsical preferences have a chance to nullify price adjustment stabilization and drive a stable economy into Chaos again.

Accordingly, it has to be concluded that in a pure exchange market economy there is no way to avoid a chaotic price movement and a global instability. We will be all of a sudden thrown into a chaotic world against a neoclassical world of a stable price mechanism. From the simulation results above, it could be even concluded that disequilibrium states are normal in a market economy! In other words, a stable price adjustment mechanism propounded by neoclassical school is rather exceptional in a market economy that is prevalently chaotic.

No one could deny this conclusion, because it is drawn from a most fundamental exchange model of a market economy. This conclusion forces us to drastically change our vision on a classical doctrine of *invisible hand* that has been believed for more than 200 years since Adam Smith.

Traditional classical and neoclassical doctrine of economics has been constructed on a linear framework of a classical Newtonian mechanics. Modern neoclassical theory of price adjustment mechanism is nothing but an application of such a classical mechanics to economics. Keynes once warned that our economic thoughts are easily enslaved by those of professional economists. It turned out that economists themselves were enslaved by classical physicists.

Modern economic theory has not only failed to provide remedies for overcoming these disequilibria caused by a chaotic market, but also has stubbornly clung, to be worse, to a traditional belief in a globally stable market economy.

Market economic analysis now has to be based on off-equilibrium transactions on Historical Time. Once economic analysis is freed from the control of *invisible hand*, market disequilibria such as recession and unemployment can be better handled on Historical Time with system dynamics method.

The MuRatopian Economy

After the collapse of the former Soviet Union in 1989, a capitalist market economy has become the only remaining alternative, no matter how violent and chaotic it is. Accordingly, free market principles are enforced globally such as market and financial deregulations, restructuring and re-engineering by business corporations, resulting in recessions and higher unemployment rates. And government tries repeatedly to exercise traditional fiscal and financial policies in vain.

In the book [78], information age is shown to be incompatible with a capitalist market economy and a mixed economy of welfare state. It then poses a necessity of new economic paradigm suitable for the information age. As one such new paradigm, I have proposed an economic system called MuRatopian economy. Interested readers are referred to "Sustainability and MuRatopian Economy" [81, Chapter 5] and "Toward A New Social Design" [78, Chapter 8].

Now that disequilibria on Historical Time are shown to be normal states in a capitalist market economy, the doctrine of Adam Smith should not be influential anymore in the information age of the 21st century. We need to change the way we think about a market economy. We have to create a new economic system that is beyond a chaotic capitalist market economy and is preferable in the new information age. This will be challenged in Part IV of chapters 12, 13, 14 and 15; that is, Macroeconomic Systems of Public Money. Specifically, chapter 15 revisits the MuRatopian economy, and incorporates it with the public money system we propose in this book as our best social design of macroeconomy for sustainable futures.

Before going so far, we have to explore how market economies and macroeconomies running on Historical Time work.

2.8 Co-Flows of Goods with Money

So far, we have focused on the attainment of the equilibrium in a market economy through price adjustment. In a market economy, however, attainment of equilibrium is necessary, but not sufficient to make transactions possible if the economy is not a so-called pure exchange economy, and it is running on historical time.

Whenever transactions are allowed at off-equilibrium prices, money as a medium of exchange has to be introduced. This is what human history tells us,

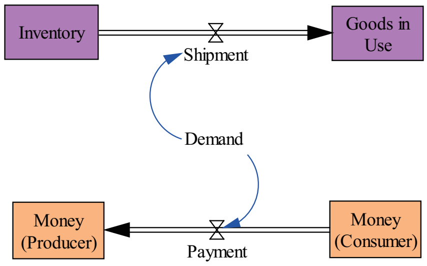

Figure 2.22: Co-flows of Goods and Money

as explored in [96]. In other words, goods and money flows simultaneously as illustrated in Figure 2.22.

Accordingly, we have to explore how to model such co-flow transactions. It will be done in the next chapter by examining accounting system.

Appendix: A Pure Exchange Economy

A Model

A pure exchange model can be represented as constituting n commodities and H consumers who own the initial endowments:

$$\bar{x}^h = (\bar{x}_1^h, \bar{x}_2^h, \ldots, \bar{x}_n^h), \quad h \in H. \tag{2.15}$$

Total supply of commodities in the economy is obtained as

$$\bar{x}_i = \sum_{h \in H} \bar{x}_i^h, \quad i = 1, 2, \ldots, n. \tag{2.16}$$

Moreover, for a given price vector $p = (p_1, p_2, \ldots, p_n)$, a consumer h's notional income is calculated as

$$I_h(p) = p\bar{x}^h = \sum_{i=1}^{n} p_i \bar{x}_i^h, \quad h \in H. \tag{2.17}$$

As a consumer h's preferences, let me assume a following Cobb-Douglas utility function in a logarithmic form where $\alpha^h > 0$:

$$u^h(x^h, \alpha^h) = \sum_{i=1}^{n} \alpha_i^h \log x_i^h. \tag{2.18}$$

It is well known that a utility function thus defined is strongly quasi-concave.

The consumer h is now assumed to seek to maximize $u^h(x^h, \alpha^h)$ subject to his budget constraint $px^h \leq I_h(p)$. Then, by a simple calculation his demand functions are obtained as

$$x_i^h(p) = \hat{\alpha}_i^h \frac{I_h(p)}{p_i}, \quad i = 1, 2, \ldots, n, \tag{2.19}$$

$$\text{where } \hat{\alpha}_i^h = \frac{\alpha_i^h}{\sum_{i=1}^{n} \alpha_i^h} \text{ and } \sum_{i=1}^{n} \hat{\alpha}_i^h = 1. \tag{2.20}$$

These non-linear demand functions are shown to be homogeneous of degree zero in price p.

Total demand for commodities is defined as

$$x_i(p) = \sum_{h \in H} x_i^h(p), \quad i = 1, 2, \ldots, n. \tag{2.21}$$

Then, excess demand functions are calculated as

$$\zeta_i(p) = \frac{1}{p_i} \sum_{h \in H} \hat{\alpha}_i^h I_h(p) - \bar{x}_i, \quad i = 1, 2, \ldots, n. \tag{2.22}$$

An equilibrium of the economy is defined to be a situation in which all markets clear for some price p^*, that is,

$$\zeta(p^*) = \{\zeta_1(p^*), \zeta_2(p^*), \dots, \zeta_n(p^*)\} = 0. \tag{2.23}$$

The existence of such an equilibrium price is reduced to find a solution in the following linear system:

$$\begin{bmatrix} a_{11} - \bar{x}_1 & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \bar{x}_2 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \bar{x}_n \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{2.24}$$

where $a_{ij} = \sum_{h \in H} \hat{\alpha}_i^h \bar{x}_j^h$.

It is shown in [35, p.100] that there exists a non-trivial positive equilibrium price $p^* \gg 0$. The existence of equilibria in an exchange economy is more generally shown by Smale[60]. Such an equilibrium price is known to be unique up to n-1 prices. This can be easily confirmed by the fact that the column sums of the above matrix are zero, or from the Walras' law: $p\zeta(p) \equiv 0$. That is to say, only relative prices are determined in the equilibrium.

Two commodities and two consumers

Let us simplify the exchange economy as consisting of two commodities and two consumers. In this simplified economy, excess demand functions are calculated as

$$\zeta_1(p) = \hat{\alpha}_1^1 \frac{I_1(p)}{p_1} + \hat{\alpha}_1^2 \frac{I_2(p)}{p_1} - \bar{x}_1, \tag{2.25}$$

$$\zeta_2(p) = \hat{\alpha}_2^1 \frac{I_1(p)}{p_2} + \hat{\alpha}_2^2 \frac{I_2(p)}{p_2} - \bar{x}_2. \tag{2.26}$$

These excess demand functions are obviously homogeneous of degree zero, and Walras' law in this economy is shown to be

$$p_1\zeta_1(p) + p_2\zeta_2(p) \equiv 0. \tag{2.27}$$

Therefore, a relative equilibrium price $p^* = (p_1^*, p_2^*)$ satisfying $\zeta_i(p^*) = 0,\ i = 1, 2$, is calculated as follows.

$$\frac{p_1^*}{p_2^*}_{|\zeta_1=0} = \frac{\hat{\alpha}_1^1 \bar{x}_2^1 + \hat{\alpha}_1^2 \bar{x}_2^2}{(1 - \hat{\alpha}_1^1)\bar{x}_1^1 + (1 - \hat{\alpha}_1^2)\bar{x}_1^2}. \tag{2.28}$$

$$\frac{p_1^*}{p_2^*}_{|\zeta_2=0} = \frac{(1 - \hat{\alpha}_2^1)\bar{x}_2^1 + (1 - \hat{\alpha}_2^2)\bar{x}_2^2}{\hat{\alpha}_2^1 \bar{x}_1^1 + \hat{\alpha}_2^2 \bar{x}_1^2}. \tag{2.29}$$

From a relation: $1 - \hat{\alpha}_1^i = \hat{\alpha}_2^i,\ i = 1, 2$, it can be shown that these two relative equilibrium prices are equal, that is,

$$\frac{p_1^*}{p_2^*}_{|\zeta_1=0} = \frac{p_1^*}{p_2^*}_{|\zeta_2=0} \tag{2.30}$$

In sum, it is demonstrated that in this simplified economy an equilibrium price exists, and only a relative price is determined, as expected from the analysis of the general model above.

Constructing Tâtonnement Processes

How can we attain an equilibrium price when it cannot be directly computed? In such a case, a tâtonnement price adjustment process is the only available method to determine an equilibrium price or even estimate it. A standard adjustment process that is often used in the literature is the following in which an adjustment coefficient λ is given exogenously.

$$p_i(t+1) = \text{Max } \{p_i(t) + \lambda\zeta_i(p(t)), 0\}, \quad i = 1, 2. \tag{2.31}$$

As an alternative tâtonnement adjustment (a), the following process is also employed:

$$p_i(t+1) = p_i(t) + \lambda\text{Max } \{\zeta_i(p(t)), 0\}, \quad i = 1, 2. \tag{2.32}$$

When prices are bounded by some minimum and maximum values, the following minmax tâtonnement adjustment (m) is occasionally applied:

$$p_i(t+1) = \text{Min } \{\bar{p}_i, \text{Max } \{p_i(t) + \lambda\zeta_i(p(t)), \underline{p}_i\}\}, \quad i = 1, 2. \tag{2.33}$$

This process is a generalization of the above standard tâtonnement adjustment process whose maximum price is assumed to be infinite.

In these processes the adjustment coefficient λ has to be arbitrarily chosen by an Auctioneer. To avoid this arbitrariness, let us construct another processes in which an adjustment coefficient λ is determined by a relative weight of prices at the iteration period t such that

$$\lambda_i(t) = \frac{p_i(t)}{\sum_{i=1}^{2} p_i(t)}, \quad i = 1, 2, \tag{2.34}$$

and call these revised coefficients composite coefficients. Thus, these composite coefficients are applied to the above three adjustment processes respectively as follows.

Standard composite tâtonnement adjustment (c)

$$p_i(t+1) = \text{Max } \{p_i(t) + \lambda_i(t)\zeta_i(p(t)), 0\}, \quad i = 1, 2. \tag{2.35}$$

Alternative composite tâtonnement adjustment (ac)

$$p_i(t+1) = p_i(t) + \lambda_i(t)\text{Max } \{\zeta_i(p(t)), 0\}, \quad i = 1, 2. \tag{2.36}$$

Minmax composite tâtonnement adjustment (mc)

$$p_i(t+1) = \text{Min } \{\bar{p}_i, \text{Max } \{p_i(t) + \lambda_i(t)\zeta_i(p(t)), \underline{p}_i\}\}, \quad i = 1, 2. \tag{2.37}$$

In this way six different tâtonnement price adjustment processes can be constructed.

Global Stability

Can any arbitrarily-chosen initial price attain an equilibrium in an exchange economy? If it can, the economy is called globally stable. Arrow, Block and Hurwicz [1] proved such a global stability under the assumptions of Walras' law, homogeneity of excess demand function and gross substitutability. Since then it has been generally adopted in the literature on microeconomics and mathematical economics, for instance [66, pp.321-329]. Walras' law and homogeneity are already shown to hold in the exchange economy. It is also shown here that for $(p_1,\ p_2) > 0$ a gross substitutability holds in the simplified economy as follows.

$$\frac{\partial \zeta_1(p)}{\partial p_2} = \frac{1}{p_1}(\hat{\alpha}_1^1 \bar{x}_2^1 + \hat{\alpha}_1^2 \bar{x}_2^2) > 0. \tag{2.38}$$

$$\frac{\partial \zeta_2(p)}{\partial p_1} = \frac{1}{p_2}(\hat{\alpha}_2^1 \bar{x}_1^1 + \hat{\alpha}_2^2 \bar{x}_1^2) > 0. \tag{2.39}$$

Equilibrium prices are attained under a condition that *an adjustment coefficient λ is fixed at its original default value*. Hence, the simplified exchange economy turned out to be globally stable.